Exercice 5 p 150

1) $\lim\limits_{1} \frac{2}{x-1}$ ∄ { gauche : $-\infty$; droite : $+\infty$

2) $\lim\limits_{-2} \frac{2-x}{2+x}$ ∄ { $-\infty$; $+\infty$

3) $\lim\limits_{0} \frac{x+1}{x^2-x}$ ∄ { $+\infty$; $-\infty$

4) $\lim\limits_{2} \frac{1}{x^2-4x+4} = +\infty$

5) $\lim\limits_{1} \frac{2x-3}{(x-1)^2} = -\infty$

6) $\lim\limits_{2} \frac{x-5}{x^2-5x+6}$ ∄ { $-\infty$; $+\infty$

7) $\lim\limits_{-3} \frac{1}{|3+x|} = +\infty$

8) $\lim\limits_{\frac{1}{3}} \frac{1}{\sqrt{1-3x}} =$ { $+\infty$; pas de sens

9) $\lim\limits_{-1} \frac{\sqrt{2-x}}{\sqrt{x+1}} =$ { pas de sens ; $+\infty$

10) $\lim\limits_{1} \frac{x+2}{\sqrt{x^2-2x+1}} = +\infty$

11) $\lim\limits_{2} \frac{x+1}{\sqrt[3]{x-2}}$ ∄ { $-\infty$; $+\infty$

Exercice 6 p 150

1) $\lim\limits_{-1} \frac{x^2+4x+3}{x^2-x-2} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{-1} \frac{(x+1)(x+3)}{(x+1)(x-2)} = \frac{-2}{3}$

2) $\lim\limits_{\frac{1}{2}} \frac{2x^2+7x-4}{4x^2-1} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{\frac{1}{2}} \frac{(2x-1)(x+4)}{(2x-1)(2x+1)} = \frac{9}{4}$

3) $\lim\limits_{2} \frac{2x^2-x-6}{x^2-4x+4} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{2} \frac{(x-2)(2x+3)}{(x-2)^2} = \lim\limits_{2} \frac{2x+3}{x-2} =$ { $-\infty$ à gauche ; $+\infty$ à droite

il y a une asymptote verticale d'équation $AV \equiv x = 2$

4) $\lim\limits_{\frac{2}{3}} \frac{3x^2-x-2}{9x^2+12x+4} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{\frac{2}{3}} \frac{(3x+2)(x-1)}{(3x+2)^2} = \lim\limits_{\frac{2}{3}} \frac{x-1}{3x+2} =$ { $+\infty$; $-\infty$

$AV \equiv x = \frac{-2}{3}$

5) $\lim\limits_{1} \frac{2x^3-x^2-4x+3}{x^2-2x+1} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{1} \frac{(x-1)^2(2x+3)}{(x-1)^2} = 5$ (Horner)

6) $\lim\limits_{2} \frac{x^3-2x^2-4x+8}{x^3-x^2-5x+6} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{2} \frac{(x-2)^2(x+2)}{(x-2)(x^2+x-3)} = 0$ (Horner)

7) $\lim\limits_{-2} \frac{3x^2+5x-2}{x^2+4x+4} \overset{0}{\underset{0}{}} \rightarrow \lim\limits_{-2} \frac{(x+2)(3x-1)}{(x+2)^2} = \lim\limits_{-2} \frac{(3x-1)}{x+2} =$ { $+\infty$; $-\infty$

$AV \equiv x = -2$

8) $\frac{7}{6}$ 9) $\frac{2}{3}$ 10) $\frac{3}{2}$ 11) $\begin{cases} -\frac{1}{3} \\ \frac{1}{3} \end{cases}$ $\lim \not\exists$

12) $\begin{cases} -4 \\ -2 \end{cases}$ $\lim \not\exists$

Exercice 7 p 151

1) $\lim\limits_{-1} \frac{x+1}{\sqrt{x+5}-2} \quad \frac{0}{0} \rightarrow \lim\limits_{-1} \frac{(x+1)(\sqrt{x+5}+2)}{x+1} = 4$

2) $\lim\limits_{\frac{1}{2}} \frac{2x-1}{\sqrt{4x+1}-\sqrt{6x}} \quad \frac{0}{0} \rightarrow \lim\limits_{\frac{1}{2}} \frac{(2x-1)(\sqrt{4x+1}+\sqrt{6x})}{-2x+1} = -2\sqrt{3}$

3) $\lim\limits_{2} \frac{\sqrt{3x+2}-\sqrt{5x-2}}{x-2} \quad \frac{0}{0} \rightarrow \lim\limits_{2} \frac{-2(x-2)}{(x-2)(\sqrt{3x+2}+\sqrt{5x-2}} = \frac{-\sqrt{2}}{4}$

4) $\lim\limits_{4} \frac{\sqrt{x+5}-3}{x-4} = \frac{1}{6}$

5) $\lim\limits_{2} \frac{\sqrt{-2x}+x}{x+2} = \frac{1}{2}$

6) $\lim\limits_{1} \frac{2-\sqrt{x+3}}{\sqrt{x-1}} \quad \frac{0}{0}$ $\begin{cases} \lim\limits_{1^-} \not\exists \text{ (domaine)} \\ \lim\limits_{1^+} = 0 \end{cases}$

7) $\lim\limits_{\frac{2}{3}} \frac{\sqrt{3x-2}}{\sqrt{6x-1}-\sqrt{5-3x}} \quad \frac{0}{0}$ $\begin{cases} \lim\limits_{\frac{2}{3}^-} \not\exists \text{ (domaine)} \\ \lim\limits_{\frac{2}{3}^+} = +\infty \end{cases}$

8) $\lim\limits_{2} \frac{x-\sqrt{3x-2}}{\sqrt{x+2}-2} \quad \frac{0}{0}$ $\lim\limits_{2} = 1$

9) $\lim\limits_{2} \frac{\sqrt{x^2-2}-\sqrt{-x}}{\sqrt{x^3+8}-\sqrt{x+2}} \quad \frac{0}{0}$ $\begin{cases} \lim\limits_{2^-} \not\exists \text{ (domaine)} \\ \lim\limits_{2^+} = 0 \end{cases}$

10) $\lim\limits_{-1} \frac{\sqrt{2-x}-\sqrt{x+4}}{\sqrt{-3x}-\sqrt{2x+5}} \quad \frac{0}{0} \quad \lim\limits_{-1} = \frac{2}{5}$

Exercice 8 p 194

1) $\lim\limits_{2} \left(\frac{x}{x-2} - \frac{x+1}{x^2-4}\right) = \begin{cases} -\infty \text{ à gauche} \\ +\infty \text{ à droite} \end{cases} \Rightarrow \lim \nexists$

$AV \equiv x = 2$

2) $\lim\limits_{-1} \left(\frac{1}{x+1} - \frac{2x}{x^2-1}\right) = \frac{1}{2}$

3) $\lim\limits_{0} \left(\frac{1}{x(1-x)} - \frac{1}{x(x+1)}\right) = 2$

4) $\lim\limits_{3} \left(\frac{1}{x+3} - \frac{1}{x-3} + \frac{6}{x^2-9}\right) = 0$

5) $\lim\limits_{1} \left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = -1$

6) $\lim\limits_{2} \left(\frac{x}{x^2-4} - \frac{1}{x+2}\right) = \begin{cases} -\infty \text{ à gauche} \\ +\infty \text{ à droite} \end{cases} \Rightarrow \lim \nexists$

$AV \equiv x = 2$

7) $\lim\limits_{0} \left(\frac{1}{x^2} - \frac{1}{x}\right) = +\infty$ $\quad AV \equiv x = 0$

Exercice 9 p 194

Après avoir trouvé un réel qui annule le dénominateur (et pas le numérateur), l'étude du signe de la fonction permet de déterminer si la limite à gauche ou à droite est $+\infty$ ou $-\infty$

1) $\text{dom } f = \mathbb{R} \setminus \{2\}$ $\quad f(2)$ de la forme $\frac{3}{0} = \infty$ signe à déterminer

x		2	
$\frac{3}{x-2}$	$-$	$\nexists$	$+$

$\lim\limits_{2^+} f(x) = +\infty$
$\lim\limits_{2^-} f(x) = -\infty$
$\Big\} \lim\limits_{2} f(x) \nexists$

$AV \equiv x = 2$

2) $\text{dom } f = \mathbb{R} \setminus \{-1, 1\}$ $\quad f(-1)$ de la forme $\frac{-3}{0}$; $f(1)$ de la forme $\frac{-1}{0}$

x		-1		1		2	
$f(x)$	$-$	$\nexists$	$+$	$\nexists$	$-$	0	$+$

$\lim\limits_{-1^-} = -\infty$
$\lim\limits_{-1^+} = +\infty$
$\Big\} \lim\limits_{-1} \nexists$

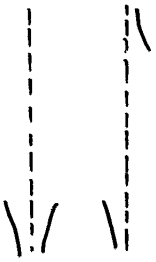
$AV_1 \equiv x = -1$
$AV_2 \equiv x = 1$

$\lim\limits_{1^-} = +\infty$
$\lim\limits_{1^+} = -\infty$
$\Big\} \lim\limits_{1} \nexists$

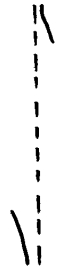
Exercice 9 p194 (suite)

3) $AV \equiv x = -2$

4) $AV_1 \equiv x = 0$
$AV_2 \equiv x = 2$

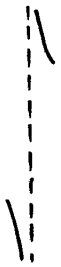


5) $AV \equiv x = 1$



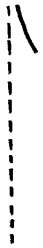
6) $AV \equiv x = 4$

7) $AV \equiv x = 2$

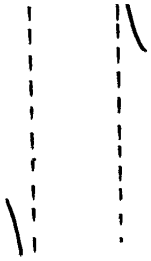


8) pas d'A.V.

9) $AV \equiv x = 0$



10) $AV_1 \equiv x = -2$
$AV_2 \equiv x = 2$



11) pas d'A.V.

12) $AV \equiv x = 2$