Equations trigonométriques

résolution graphique (géogébra)

- poser et dessiner $f(x)$ = membre de gauche
  $g(x)$ = membre de droite
- matérialiser les points d'intersection des 2 courbes
- les solutions sont les abscisses des points d'intersection

Si le membre de droite est 0, $f(x)$ = membre de gauche et les solutions sont les racines (intersections avec l'axe Ox)

résolution algébrique

- transformer l'équation jusqu'à obtention d'une comparaison entre 2 sinus, 2 cosinus ou 2 tangentes.
- utiliser ensuite les égalités entre les angles associés (facile avec un cercle trigonométrique)

Ex 7 p 103-104 (sol particulières dans $[0, 2\pi[$)

1) $\cos x = \cos \frac{\pi}{3} \Leftrightarrow x = \pm \frac{\pi}{3} + k.2\pi$ $\quad \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

2) $\sin x = \sin \frac{5\pi}{4} \Leftrightarrow x = \frac{5\pi}{4} + k2\pi$ ou $x = \frac{7\pi}{4} + k.2\pi$ $\quad \left\{\frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

3) $\tg x = \tg \frac{3\pi}{4} \Leftrightarrow x = \frac{3\pi}{4} + k.\pi$ $\quad \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

4) $\sin 2x = \frac{-1}{2} \Leftrightarrow \sin 2x = \sin \frac{7\pi}{12}$

$\Leftrightarrow x = \frac{7\pi}{12} + k\pi$ ou $x = \frac{11\pi}{12} + k\pi$

$\left\{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

5) $\tg 4x = \sqrt{3} \Leftrightarrow \tg 4x = \tg \frac{\pi}{3} \Leftrightarrow 4x = \frac{\pi}{3} + k\pi$

$\Leftrightarrow x = \frac{\pi}{12} + k\frac{\pi}{4}$ $\quad \left\{\frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{6}, \frac{13\pi}{12}, \frac{4\pi}{3}, \frac{19\pi}{12}, \frac{11\pi}{6}\right\}$

6) $\cos 5x = \frac{-\sqrt{3}}{2} \Leftrightarrow \cos 5x = \cos \frac{5\pi}{6} \Leftrightarrow x = \pm \frac{\pi}{6} + k. \frac{2\pi}{5}$

$\left\{ \frac{\pi}{6}, \frac{7\pi}{30}, \frac{17\pi}{30}, \frac{19\pi}{30}, \frac{29\pi}{30}, \frac{31\pi}{30}, \frac{41\pi}{30}, \frac{43\pi}{30}, \frac{53\pi}{30}, \frac{11\pi}{6} \right\}$

7) $\cos\left(3x + \frac{\pi}{2}\right) = \frac{-1}{2} \Leftrightarrow \cos\left(3x + \frac{\pi}{2}\right) = \cos \frac{2\pi}{3}$

$\Leftrightarrow x = \frac{\pi}{18} + k. \frac{2\pi}{3}$ ou $x = \frac{5\pi}{18} + k. \frac{2\pi}{3}$

$\left\{ \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18} \right\}$

8) $\sin\left(\frac{\pi}{6} - 2x\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(\frac{\pi}{6} - 2x\right) = \sin \frac{\pi}{3}$

$\left\{ \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{7\pi}{4}, \frac{23\pi}{12} \right\}$

9) $\operatorname{tg}\left(2x - \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{3} \Leftrightarrow \operatorname{tg}\left(2x - \frac{\pi}{3}\right) = \operatorname{tg} \frac{-\pi}{6}$ CE $x \neq \frac{\pi}{3} + k. \frac{\pi}{2}$

$\left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

10) $\operatorname{tg} x = \sqrt{3}$ ou $\operatorname{tg} x = -\sqrt{3} \Leftrightarrow \operatorname{tg} x = \operatorname{tg} \frac{\pi}{3}$ ou $\operatorname{tg} x = \operatorname{tg} \frac{-\pi}{3}$

$\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$ CE $x \neq \frac{\pi}{2} + k\pi$

11) $x = \frac{5\pi}{48} + k\frac{\pi}{2}$ ou $x = \frac{13\pi}{24} + k\pi$

$\left\{ \frac{5\pi}{48}, \frac{13\pi}{24}, \frac{29\pi}{48}, \frac{37\pi}{24}, \frac{53\pi}{48}, \frac{77\pi}{48} \right\}$

12) $x = k\pi$ ou $x = \frac{\pi}{12} + k\frac{\pi}{6}$

$\left\{ 0, \frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{11\pi}{12}, \pi, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{3}, \frac{23\pi}{12} \right\}$

13) CE $x \neq \frac{\pi}{9} + k.\frac{\pi}{3}$ et $x \neq \frac{\pi}{12} + k.\frac{\pi}{2}$

$x = \frac{\pi}{10} + k.\frac{\pi}{5}$ $\left\{\frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}, \frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}\right\}$

14) $\sin 3x = \sin\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - x\right)\right)$ $x = \frac{5\pi}{24} + k\frac{\pi}{2}$ ou $x = \frac{\pi}{12} + k\pi$

$\left\{\frac{\pi}{12}, \frac{5\pi}{24}, \frac{17\pi}{12}, \frac{13\pi}{12}, \frac{29\pi}{12}, \frac{41\pi}{12}\right\}$

15) $\cos 3x = \sin(-x) \Leftrightarrow \cos 3x = \cos\left(\frac{\pi}{2} + x\right)$

$x = \frac{\pi}{4} + k\pi$ ou $x = \frac{-\pi}{8} + k\frac{\pi}{2}$

$\left\{\frac{\pi}{4}, \frac{3\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\right\}$

16) $\sin 2x = \sin\left(\frac{\pi}{2} - 3x\right)$ $x = \frac{\pi}{10} + k\frac{2\pi}{5}$ ou $x = \frac{-\pi}{2} + k.2\pi$

$\left\{\frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}\right\}$

17) $\sin 4x = \sin(-x)$ $x = k\frac{2\pi}{5}$ ou $x = \frac{\pi}{3} + k.\frac{2\pi}{3}$

$\left\{0, \frac{\pi}{3}, \frac{2\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{5\pi}{3}, \frac{8\pi}{5}\right\}$

18) $\sin\left(3x + \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2} - \left(x - \frac{\pi}{3}\right)\right)$ $x = \frac{\pi}{6} + k\frac{\pi}{2}$ ou $x = k\pi$

$\left\{0, \frac{\pi}{6}, \frac{2\pi}{3}, \pi, \frac{7\pi}{6}, \frac{5\pi}{3}\right\}$

19) CE $x \neq \frac{-\pi}{12} + k\frac{\pi}{2}$ et $x \neq \frac{\pi}{6} + k\frac{\pi}{3}$

$\tan\left(\frac{\pi}{3} - 2x\right) = \tan(-3x)$ $x = \frac{-\pi}{3} + k\pi$

$\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

20) $4\sin^2 x = 3 \iff \sin^2 x = \frac{3}{4}$

$\iff \sin x = \frac{\sqrt{3}}{2}$ ou $\sin x = \frac{-\sqrt{3}}{2} \iff x = \frac{-\pi}{3} + k\pi$

$\left\{ \frac{2\pi}{3}, \frac{5\pi}{3} \right\}$

21) $\cos^2 x = \frac{1}{2} \iff \cos x = \frac{\sqrt{2}}{2}$ ou $\cos x = -\frac{\sqrt{2}}{2}$

$x = \pm \frac{\pi}{4} + k\pi$ $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

22) $\sin^3 x - \sin x = 0 \iff \sin x (\sin^2 x - 1) = 0$

$\iff \sin x (\sin x - 1)(\sin x + 1) = 0$

$\iff \sin x = 0$ ou $\sin x = 1$ ou $\sin x = -1$

$x = k \cdot \frac{\pi}{2}$ $\left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$

23) On a $x \neq k\pi$

$\frac{\cos^2 x}{\sin^2 x} = 1 \iff \cos^2 x = \sin^2 x$

$\iff \cos^2 x - \sin^2 x = 0$

$\iff \cos 2x = 0$

$x = \frac{\pi}{4} + k\pi$ ou $x = -\frac{\pi}{4} + k\pi$ $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

24) voir 23.

Exercice 13 p105

On ne détaillera ici que la transformation de l'équation initiale en équation(s) élémentaire(s).

1) $2\cos 2x - 4\cos x + 3 = 0$

$2(\cos^2 x - \sin^2 x) - 4\cos x + 3 = 0$

$2\cos^2 x - 2\sin^2 x - 4\cos x + 3 = 0$

$2\cos^2 x - 2(1 - \cos^2 x) - 4\cos x + 3 = 0$

$4\cos^2 x - 4\cos x + 1 = 0$ second degré en $\cos x$

$\Delta = 0$

$\cos x = \frac{4}{8} = \frac{1}{2}$

2) $\sin 2x = 2\sin x$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$\sin x = 0$ ou $\cos x = 1$

3) $\cos 2x - 2 = 3\cos x$

$\cos^2 x - \sin^2 x - 2 - 3\cos x = 0$

$\cos^2 x - (1 - \cos^2 x) - 2 - 3\cos x = 0$

$2\cos^2 x - 3\cos x - 3 = 0$ second degré en $\cos x$

$\Delta = 33$

$\cos x = \frac{3 - \sqrt{33}}{4}$ ou $\cos x = \frac{3 + \sqrt{33}}{4} > 1$

$\cos x = -0,6861406...$ impossible

$\cos x = \cos 2,3269668...$

4) $4\cos 2x + 4\sin^2 x - 3 = 0$

$4(\cos^2 x - \sin^2 x) + 4\sin^2 x - 3 = 0$

$4\cos^2 x - 4\sin^2 x + 4\sin^2 x - 3 = 0$

$4\cos^2 x - 3 = 0$ $\cos^2 x = \frac{3}{4}$ $\cos x = \frac{\sqrt{3}}{2}$ ou $\cos x = \frac{-\sqrt{3}}{2}$

5) $\cos 2x + \sin x = 0$

$\cos 2x = -\sin x$

$\cos 2x = \sin(-x)$

$\cos 2x = \cos\left(\frac{\pi}{2} - (-x)\right)$

$\cos 2x = \cos\left(\frac{\pi}{2} + x\right)$

6) $\cos x \cos 3x - \sin x \sin 3x = -1$

$\cos(x + 3x) = -1$

$\cos 4x = -1$

7) $\cos 2x = \cos x + 1$

$\cos^2 x - \sin^2 x - \cos x - 1 = 0$

$\cos^2 x - (1 - \cos^2 x) - \cos x - 1 = 0$

$2\cos^2 x - \cos x - 2 = 0$ second degré en $\cos x$

$\Delta = 17$

$\cos x = \frac{1 + \sqrt{17}}{4} > 1$ ou $\cos x = \frac{1 - \sqrt{17}}{4} = -0{,}78077\ldots$

impossible $\cos x = \cos 2{,}4667\ldots$

8) $\sqrt{2}\cos x = \tan x$ CE : $x \neq \frac{\pi}{2} + k\pi$

$\sqrt{2}\cos x = \frac{\sin x}{\cos x}$

$\sqrt{2}.\cos^2 x = \sin x$

$\sqrt{2}(1 - \sin^2 x) - \sin x = 0$

$\sqrt{2} - \sqrt{2}.\sin^2 x - \sin x = 0$ second degré en $\sin x$

$\Delta = 9$

$\sin x = \frac{1 + 3}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$ ou $\sin x = \frac{1 - 3}{2\sqrt{2}} = \frac{-\sqrt{2}}{2}$

impossible.

9) $\sin 2x = \cos x$

$\sin 2x = \sin\left(\frac{\pi}{2} - x\right)$

10) $2\sin^2 x + \sin^2 2x = 2$

$2\sin^2 x - (2\sin x \cos x)^2 - 2 = 0$

$2\sin^2 x - 4\sin^2 x \cos^2 x - 2 = 0$

$2\sin^2 x - 4\sin^2 x (1 - \sin^2 x) - 2 = 0$

$2\sin^2 x - 4\sin^2 x + 4\sin^4 x - 2 = 0$

$-2\sin^2 x + 4\sin^4 x - 2 = 0$

$2(2\sin^4 x - \sin^2 x - 1) = 0$ second degré en $\sin^2 x$

$\Delta = 9$

$\sin^2 x = \frac{1+3}{4} = 1$ ou $\sin^2 x = \frac{1-3}{4}$ impossible

$\sin x = 1$ ou $\sin x = -1$

11) $\sin 4x - 4\sin x \cos^2 x = 0$

$\sin 2 \cdot 2x - 4\sin x \cos^2 x = 0$

$2\sin 2x \cos 2x - 4\sin x \cos^2 x = 0$

$2 \cdot 2 \cdot \sin x \cos x \cos 2x - 4\sin x \cos^2 x = 0$

$4\sin x \cos x \cos 2x - 4\sin x \cos^2 x = 0$

$4\sin x \cos x (\cos 2x - \cos x) = 0$

$\sin x = 0$ ou $\cos x = 0$ ou $\cos 2x = \cos x$

12) $\sin x \cos\left(2x + \frac{\pi}{3}\right) + \sin\left(2x + \frac{\pi}{3}\right)\cos x = -\frac{1}{2}$

$\sin\left(x + 2x + \frac{\pi}{3}\right) = \frac{-1}{2}$

$\sin\left(3x + \frac{\pi}{3}\right) = \frac{-1}{2}$

13) $\tan x - \sin x = \sin 2x$ $\quad$ ce $\in$ $x \neq \frac{\pi}{2} + k\pi$

$$\frac{\sin x}{\cos x} - \sin x = \sin 2x$$

$$\frac{\sin x - \sin x \cos x}{\cos x} = \sin 2x$$

$$\sin x - \sin x \cos x = \cos x \cdot 2 \sin x \cos x$$
$$\sin x - \sin x \cos x - 2 \sin x \cos^2 x = 0$$
$$\sin x \left(1 - \cos x - 2\cos^2 x\right) = 0$$
$\sin x = 0$ ou $-2\cos^2 x - \cos x + 1 = 0$ 2e degré en $\cos x$

$\Delta = 9$

$\cos x = \frac{1+3}{-4} = -1$ ou $\cos x = \frac{1-3}{-4} = \frac{1}{2}$

14) $\cos^2 2x - 3\sin^2 x + 2 = 0$
$$\left(\cos^2 x - \sin^2 x\right)^2 - 3\sin^2 x + 2 = 0$$
$$\left(1 - \sin^2 x - \sin^2 x\right)^2 - 3\sin^2 x + 2 = 0$$
$$\left(1 - 2\sin^2 x\right)^2 - 3\sin^2 x + 2 = 0$$
$$\left(1 - 4\sin^2 x + 4\sin^4 x\right) - 3\sin^2 x + 2 = 0$$
$4\sin^4 x - 7\sin^2 x + 3 = 0$ 2e degré en $\sin^2 x$

$\Delta = 1$

$\sin^2 x = \frac{7+1}{8} = 8$ ou $\sin^2 x = \frac{7-1}{8} = \frac{3}{4}$

impossible $\quad$ $\sin x = \frac{\sqrt{3}}{2}$ ou $\sin x = \frac{-\sqrt{3}}{2}$