QUESTION 1

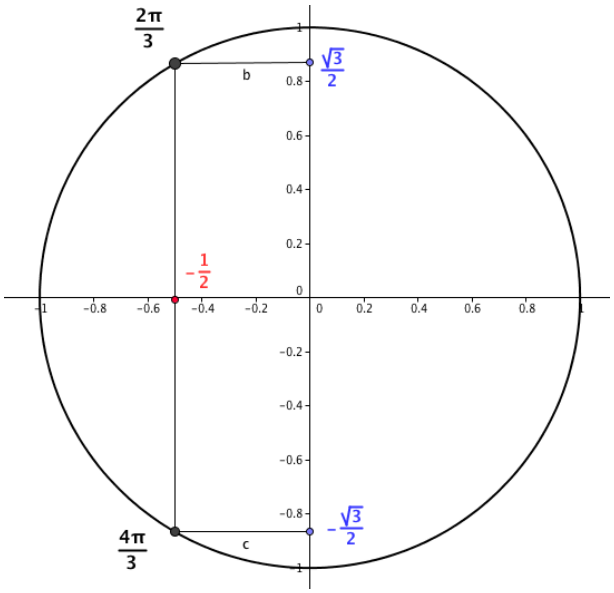
Voir notes de cours.

QUESTION 2

(1) $\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
$\cos(2a) = \cos(a+a) = \cos a \cdot \cos a - \sin a \cdot \sin a = \cos^2 a - \sin^2 a$

(2) $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \cdot \tan b}$
$\tan(a+a) = \dfrac{\tan a + \tan a}{1 - \tan a \cdot \tan a} = \dfrac{2\tan a}{1 - \tan^2 a}$

QUESTION 3



$$\sin x + \sin\left(x + \frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3} + x\right) =$$
$$\sin x + \sin x \cos\frac{2\pi}{3} + \cos x \sin\frac{2\pi}{3} + \sin x \cos\frac{4\pi}{3} + \cos x \sin\frac{4\pi}{3} =$$
$$\sin x + \sin x\left(\frac{-1}{2}\right) + \cos x\left(\frac{\sqrt{3}}{2}\right) + \sin x\left(\frac{-1}{2}\right) + \cos x\left(\frac{-\sqrt{3}}{2}\right) = 0$$

QUESTION 4

$$\cos a = -\sqrt{1 - \left(\frac{-3}{4}\right)^2} = -\sqrt{1 - \left(\frac{9}{16}\right)} = -\sqrt{\frac{7}{16}} = \frac{-\sqrt{7}}{4}$$

$$\sin b = -\sqrt{1 - \left(\frac{9}{10}\right)^2} = -\sqrt{1 - \left(\frac{81}{100}\right)} = -\sqrt{\frac{19}{100}} = \frac{-\sqrt{19}}{10}$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$= \frac{-\sqrt{7}}{4} \cdot \frac{9}{10} + \frac{-3}{4} \cdot \frac{-\sqrt{19}}{10} = \frac{3\sqrt{19} - 9\sqrt{7}}{40}$$

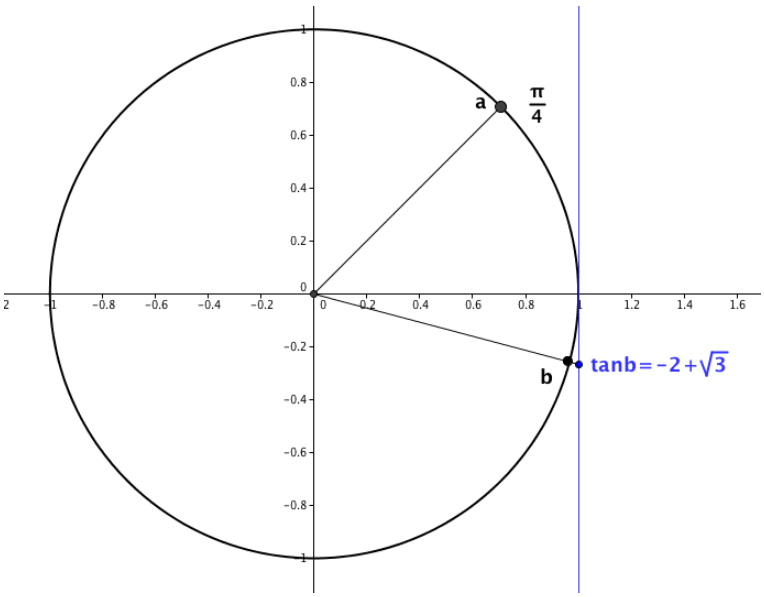
QUESTION 5

$\cos(45^\circ + a) - \sin(a - 135^\circ) =$
$\cos 45^\circ \cos a - \sin 45^\circ \sin a - (\sin a \cos 135^\circ - \cos a \sin 135^\circ) =$

$$\sin 135^\circ = \frac{\sqrt{2}}{2} \text{ et } \cos 135^\circ = -\frac{\sqrt{2}}{2} \text{ (voir cercle trigonométrique)}$$

$$\frac{\sqrt{2}}{2}\cos a - \frac{\sqrt{2}}{2}\sin a + \frac{\sqrt{2}}{2}\sin a + \frac{\sqrt{2}}{2}\cos a = \sqrt{2} \cdot \cos a$$

QUESTION 6



(1) $a = \dfrac{\pi}{4}(45^\circ) \Rightarrow \tan a = 1$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b} = \frac{1 - (\sqrt{3}-2)}{1 + 1 \cdot (\sqrt{3}-2)} = \frac{3-\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} = \sqrt{3}$$

$a - b = 60^\circ$

(2) $b = -15^\circ$

QUESTION 7

(1) $$\frac{\tan x - \tan y}{\tan x + \tan y} = \frac{\dfrac{\sin x}{\cos x} - \dfrac{\sin y}{\cos y}}{\dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}} = \frac{\dfrac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}}{\dfrac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}} = \frac{\sin(x-y)}{\sin(x+y)}$$

(2) $\sin(a-b)\cos b + \sin b \cos(a-b) = \sin a$
$(\sin a \cos b - \sin b \cos a)\cos b + \sin b(\cos a \cos b + \sin a \sin b) = \sin a$
$\sin a \cos^2 b - \sin a \cos a \cos b + \sin b \cos a \cos b + \sin a \sin^2 b = \sin a$
$\sin a \cos^2 b + \sin a \sin^2 b = \sin a$
$\sin a(\cos^2 b + \sin^2 b) = \sin a$

ou, mieux, *est de la forme* $\sin X \cos Y + \cos X \sin Y = \sin(X+Y)$ (...YO)

$\sin(a - b + b) = \sin a$

(3) $\sin(4x + \frac{3\pi}{4})\cos(2x + \frac{3\pi}{4}) - \cos(4x + \frac{3\pi}{4})\sin(2x + \frac{3\pi}{4})$

est de la forme $\sin a \cos b - \cos a \sin b = \sin(a-b)$ (...YO)

$$\sin(4x + \tfrac{3\pi}{4})\cos(2x + \tfrac{3\pi}{4}) - \cos(4x + \tfrac{3\pi}{4})\sin(2x + \tfrac{3\pi}{4}) = \sin\Big((4x + \tfrac{3\pi}{4}) - (2x + \tfrac{3\pi}{4})\Big) =$$

$\sin 2x = 2 \sin x \cos x$

QUESTION 8

$\sin(x + \frac{\pi}{3}) = \sin(2x - \frac{\pi}{4})$

angles égaux

$$x + \frac{\pi}{3} = 2x - \frac{\pi}{4} + k2\pi$$
$$-x = -\frac{7\pi}{12} + k2\pi$$
$$x = \frac{7\pi}{12} + k2\pi$$
$$k = 0 \to x = \frac{7\pi}{12}$$

ou

angles supplémentaires

$$x + \frac{\pi}{3} = \pi - 2x + \frac{\pi}{4} + k2\pi$$
$$3x = \frac{11\pi}{12} + k2\pi$$
$$x = \frac{11\pi}{36} + \frac{2k\pi}{3}$$
$$k = 0 \to x = \frac{11\pi}{36}$$
$$k = 1 \to x = \frac{35\pi}{36}$$
$$k = 2 \to x = \frac{59\pi}{36}$$

$$Sol = \left\{ \frac{7\pi}{12}, \frac{11\pi}{36}, \frac{35\pi}{36}, \frac{59\pi}{36} \right\}$$